GEOMETRY.

312. Proposed by F. H. SAFFORD, Ph. D., The University of Pennsylvania, Philadelphia, Pa.

A variable circle passes through a fixed point and is tangent to a given circle. If a diameter of the first circle passes through the fixed point find the locus of its other extremity.

Solution by G. W. GREENWOOD, M. A. (Oxon), Roanoke College, Salem, Va.

Since the circle of a radius vector of a central conic is always tangent to its auxiliary circle, the required locus is the conic having the given circle as auxiliary circle and the given point as one focus.

Or, the following method:

Call the center of the given circle 0; its radius, a; the given point, S; the point symmetrical to S with respect to 0, S'; the radius of the circle through S tangent to the given circle, r; the other extremity of the diameter through S, P; the mid-point of SP (the center of the tangent circle), A. Suppose that S is *without* the circle. If the circles are tangent externally, we have

$$PS'-PS=2(AO-AS)=2(AO-r)=2a.$$

If the circles are tangent internally, we get

$$PS-PS'=2a.$$

Therefore, the locus of P is an hyperbola.

Next, suppose that S is *within* the given circle. We get

$$PS'+PS=2(AO+r)=2a.$$

Therefore, the locus of P is an ellipse.

In either case the conic has the points S and S' as foci.

The cases in which S coincides with the center of the given circle, or is on the given circle, may easily be dealt with independently.

Also solved by G. B. M. Zerr, J. Scheffer, C. N. Schmall, and J. S. Brown.

313. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Prove that an algebraic curve of odd degree which is symmetrical with respect to a center has the center on the curve.

No solution has been received.

314. Proposed by F. ANDEREGG, A. M., Professor of Mathematics, Oberlin College, Oberlin, Ohio.

Find the area of the triangle bounded by the lines $l\alpha+m\beta+n\gamma=0$; $l'\alpha+m'\beta+n'\gamma=0$; $l''\alpha+m''\beta+n''\gamma=0$, where α stands for $x\cos\alpha+y\sin\alpha-p$, etc. [See Salmon's *Conic Sections*, 6th ed., p. 130, Ex. 1.]